As we' ll derive in recitation, the equation for the current I(t) in terms of the potential V(t) is the second-order linear constant coefficient equation

I''(t) + (R/L) I'(t) + (1/CL) I(t) = V'(t)

where R, C and L are the resistance, capacitance, and inductance respectively.

Let's look at this in a few simple cases. We'll start with V'(t) = 0, so we have a constant potential established. Think of a battery running the circuit. Then our equation reduces to

I''(t) + (R/L) I'(t) + (1/CL) I(t) = 0

In the case when the resistance is zero (an idealized case where we have no loss due to heat), this simplifies even further to

I''(t) + (1/CL) I(t) = 0

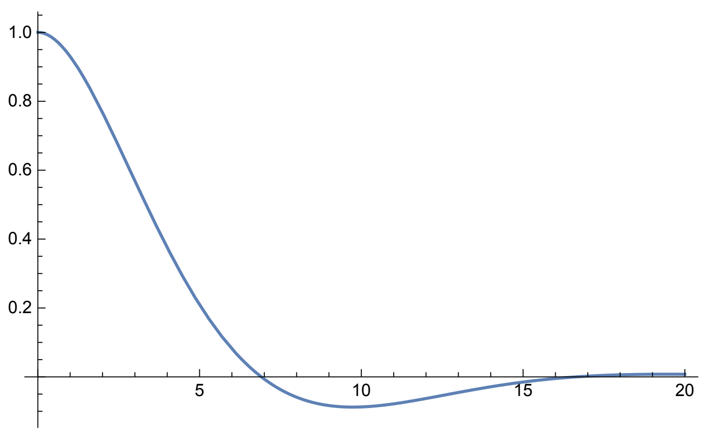
This should look very familiar - it's the equation for oscillatory motion. Its plot is sinusoidal and corresponds to oscillatory motion of the current, sloshing back and forth through the circuit. Here are some examples; in the first, R = 1, L = 2, C = 3; in the second, R = 1/10.

In[6]:=
```
DSolve[{J''[t] + (1/2) J'[t] + (1/6) J[t] == 0, J[0] == 1, J'[0] == 0}, J, t]
```

Out[6]=
$$\left\{\left\{J \to \text{Function}\left[\{t\}, \frac{1}{5} e^{-t/4} \left(5 \,\text{Cos}\left[\frac{1}{4}\sqrt{\frac{5}{3}}\, t\right] + \sqrt{15}\, \text{Sin}\left[\frac{1}{4}\sqrt{\frac{5}{3}}\, t\right]\right)\right]\right\}\right\}$$

In[8]:=
$$\text{Plot}\left[\frac{1}{5} e^{-t/4} \left(5 \,\text{Cos}\left[\frac{1}{4}\sqrt{\frac{5}{3}}\, t\right] + \sqrt{15}\, \text{Sin}\left[\frac{1}{4}\sqrt{\frac{5}{3}}\, t\right]\right), \{t, 0, 20\}\right]$$

Out[8]=



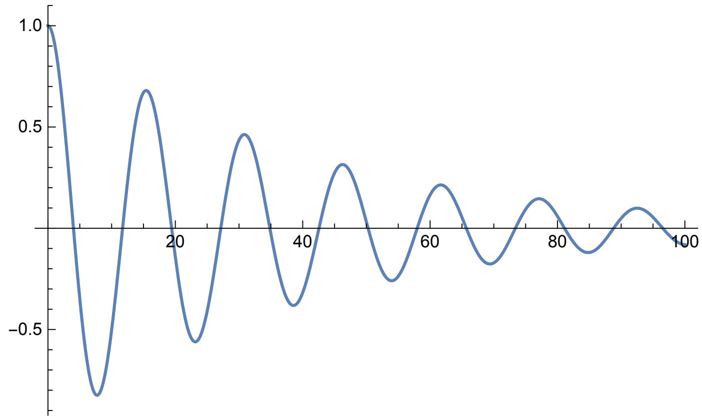

```
(* Example of large resistance and lots of damping. *)
```

In[9]:=
```
DSolve[{J''[t] + (1/20) J'[t] + (1/6) J[t] == 0, J[0] == 1, J'[0] == 0}, J, t]
```

Out[9]=
$$\left\{\left\{J \to \text{Function}\left[\{t\}, \frac{1}{797} e^{-t/40} \left(797 \,\text{Cos}\left[\frac{1}{40}\sqrt{\frac{797}{3}}\, t\right] + \sqrt{2391}\, \text{Sin}\left[\frac{1}{40}\sqrt{\frac{797}{3}}\, t\right]\right)\right]\right\}\right\}$$

In[11]:= $$\text{Plot}\left[\frac{1}{797}\, e^{-t/40}\left(797\, \text{Cos}\left[\frac{1}{40}\sqrt{\frac{797}{3}}\, t\right] + \sqrt{2391}\, \text{Sin}\left[\frac{1}{40}\sqrt{\frac{797}{3}}\, t\right]\right),\ \{t,\ 0,\ 100\}\right]$$

Out[11]=



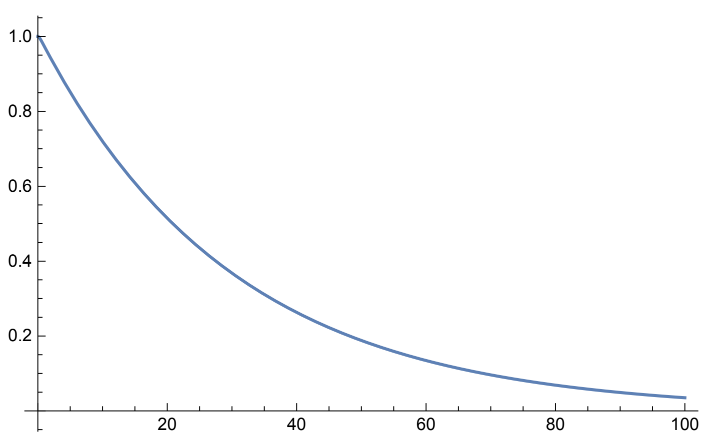

```
(* Smaller resistance *)
```

In[12]:= `DSolve[{J''[t] + (10/2) J'[t] + (1/6) J[t] == 0, J[0] == 1, J'[0] == 0}, J, t]`

Out[12]= $$\left\{\left\{J \to \text{Function}\left[\{t\},\ \frac{1}{146}\left(73\, e^{\left(-\frac{5}{2}-\frac{\sqrt{\frac{73}{3}}}{2}\right)t} - 5\sqrt{219}\, e^{\left(-\frac{5}{2}-\frac{\sqrt{\frac{73}{3}}}{2}\right)t} + 73\, e^{\left(-\frac{5}{2}+\frac{\sqrt{\frac{73}{3}}}{2}\right)t} + 5\sqrt{219}\, e^{\left(-\frac{5}{2}+\frac{\sqrt{\frac{73}{3}}}{2}\right)t}\right)\right]\right\}\right\}$$

In[15]:= $$\text{Plot}\left[\frac{1}{146}\left(73\, e^{\left(-\frac{5}{2}-\frac{\sqrt{\frac{73}{3}}}{2}\right)t} - 5\sqrt{219}\, e^{\left(-\frac{5}{2}-\frac{\sqrt{\frac{73}{3}}}{2}\right)t} + 73\, e^{\left(-\frac{5}{2}+\frac{\sqrt{\frac{73}{3}}}{2}\right)t} + 5\sqrt{219}\, e^{\left(-\frac{5}{2}+\frac{\sqrt{\frac{73}{3}}}{2}\right)t}\right),\ \{t,\ 0,\ 100\}\right]$$

Out[15]=



```
(* Huge resistance - overdamping *)
```

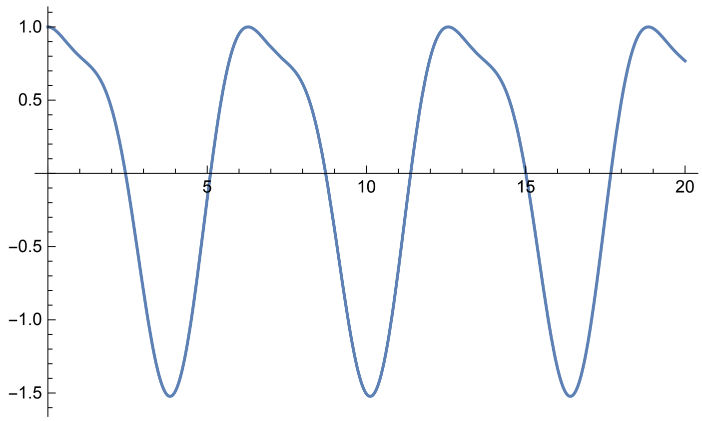
Now let' s look at the case where we introduce a non-zero voltage, like an oscillator. For simplicity, let's take R = 0 to start and use a pure oscillator like V(t) = sin(2t).

```
In[16]:= DSolve[{J''[t] + (0) J'[t] + J[t] == Sin[2 t], J[0] == 1, J'[0] == 0}, J, t]

Out[16]= {{J → Function[{t},
            1/6 (6 Cos[t] + 4 Sin[t] - 3 Cos[t] Sin[t] - Cos[3 t] Sin[t] - 4 Cos[t] Sin[t]^3)]}}

In[17]:= Plot[
          1/6 (6 Cos[t] + 4 Sin[t] - 3 Cos[t] Sin[t] - Cos[3 t] Sin[t] - 4 Cos[t] Sin[t]^3), {t, 0, 20}]

Out[17]=
```

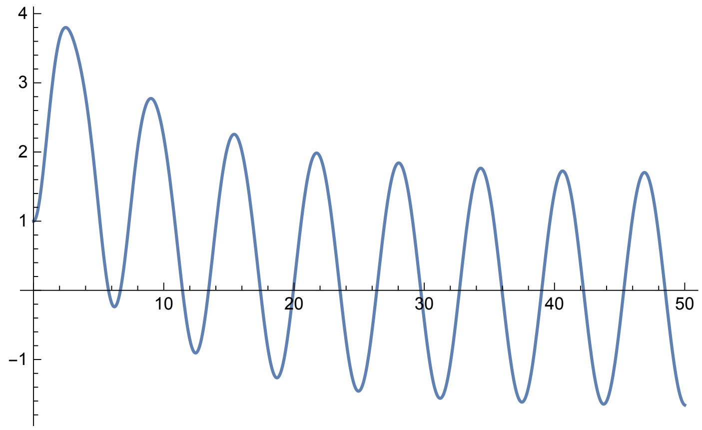



```
In[37]:= DSolve[
          {J''[t] + (0) J'[t] + J[t] == (Cos[2 t] + 3) E^(-t/10), J[0] == 1, J'[0] == 0}, J, t]

Out[37]= {{J → Function[{t},
            -1/91001 e^(-t/10) (149399 e^(t/10) Cos[t] - 225250 Cos[t]^2 - 15150 Cos[t] Cos[3 t] -
               32040 e^(t/10) Sin[t] + 9010 Cos[t] Sin[t] + 505 Cos[3 t] Sin[t] -
               315350 Sin[t]^2 - 505 Cos[t] Sin[3 t] - 15150 Sin[t] Sin[3 t])]}}
```

In[39]:=
```
Plot[-1/91001 e^(-t/10) (149399 e^(t/10) Cos[t] - 225250 Cos[t]^2 - 15150 Cos[t] Cos[3 t] -
    32040 e^(t/10) Sin[t] + 9010 Cos[t] Sin[t] + 505 Cos[3 t] Sin[t] -
    315350 Sin[t]^2 - 505 Cos[t] Sin[3 t] - 15150 Sin[t] Sin[3 t]), {t, 0, 50}]
```

Out[39]=



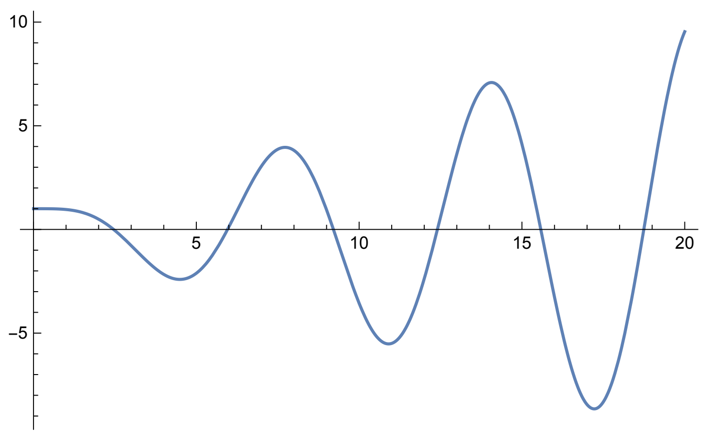

```
(* Here, the oscillation does some funny stuff at the beginning,
and then dies out as time goes on -
 we return to a purely oscillatory solution *)
```

In[40]:=
```
DSolve[{J''[t] + (0) J'[t] + J[t] == Cos[t], J[0] == 1, J'[0] == 0}, J, t]
```

Out[40]=
```
{{J → Function[{t}, 1/4 (2 Cos[t] + 2 Cos[t]^3 + 2 t Sin[t] + Sin[t] Sin[2 t])]}}
```

In[43]:=
```
Plot[1/4 (2 Cos[t] + 2 Cos[t]^3 + 2 t Sin[t] + Sin[t] Sin[2 t]), {t, 0, 20}]
```

Out[43]=

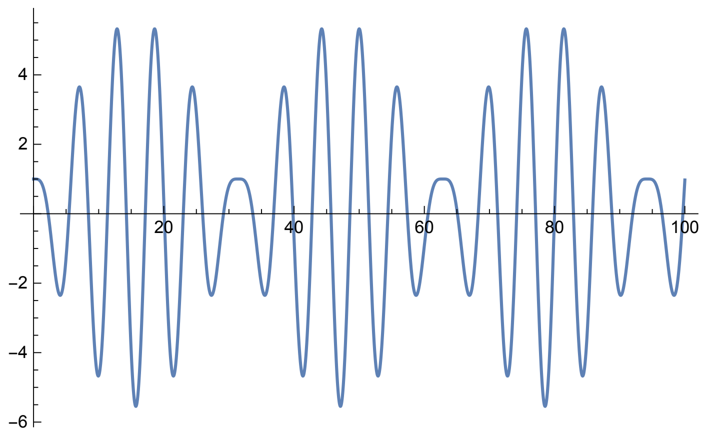



```
(* Here we have resonance -
 the current feeds back and increases itself over time, blowing up in strength *)
```

In[46]:=
```
DSolve[{J''[t] + (0) J'[t] + J[t] == Cos[(120/100) t], J[0] == 1, J'[0] == 0}, J, t]
```

Out[46]=
$$\left\{\left\{J \to \text{Function}\left[\{t\}, \frac{1}{22}\left(72\,\text{Cos}[t] - 55\,\text{Cos}\left[\frac{t}{5}\right]\text{Cos}[t] + 5\,\text{Cos}[t]\,\text{Cos}\left[\frac{11\,t}{5}\right] + 55\,\text{Sin}\left[\frac{t}{5}\right]\text{Sin}[t] + 5\,\text{Sin}[t]\,\text{Sin}\left[\frac{11\,t}{5}\right]\right)\right]\right\}\right\}$$

In[47]:=
$$\text{Plot}\left[\frac{1}{22}\left(72\,\text{Cos}[t] - 55\,\text{Cos}\left[\frac{t}{5}\right]\text{Cos}[t] + 5\,\text{Cos}[t]\,\text{Cos}\left[\frac{11\,t}{5}\right] + 55\,\text{Sin}\left[\frac{t}{5}\right]\text{Sin}[t] + 5\,\text{Sin}[t]\,\text{Sin}\left[\frac{11\,t}{5}\right]\right), \{t, 0, 100\}\right]$$




```
(* Here we see beats that continue
 periodically. Here's another plot over long time: *)
```

In[49]:=
$$\text{Plot}\left[\frac{1}{22}\left(72\,\text{Cos}[t] - 55\,\text{Cos}\left[\frac{t}{5}\right]\text{Cos}[t] + 5\,\text{Cos}[t]\,\text{Cos}\left[\frac{11\,t}{5}\right] + 55\,\text{Sin}\left[\frac{t}{5}\right]\text{Sin}[t] + 5\,\text{Sin}[t]\,\text{Sin}\left[\frac{11\,t}{5}\right]\right), \{t, 0, 500\}\right]$$

Out[49]=